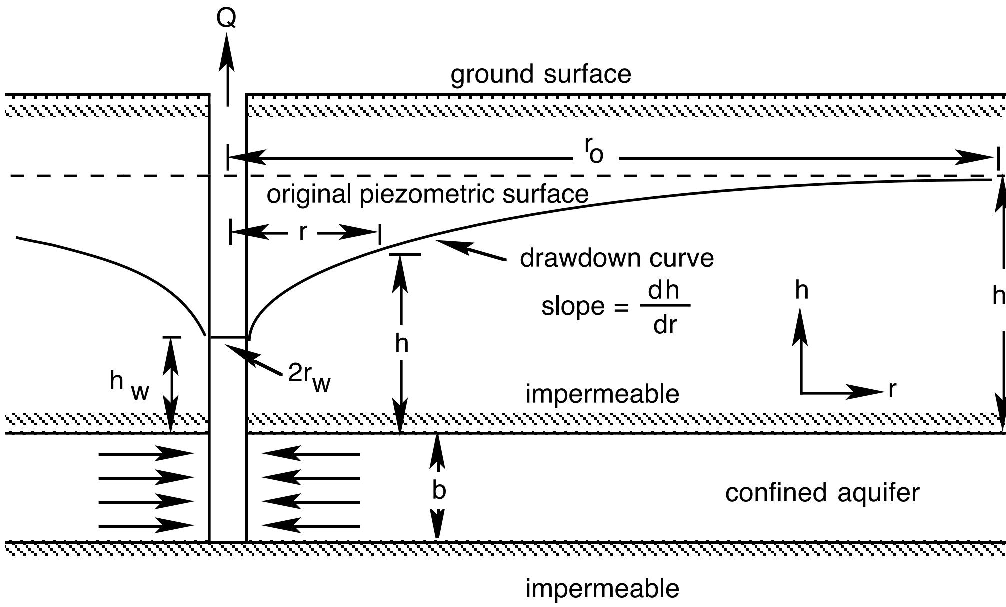
WELL FLOW EQUATIONS

1. HOMOGENEOUS ISOTROPIC CONFINED AQUIFER, STEADY-STATE FLOW, FULLY-PENETRATING WELL



b	thickness of aquifer
h	elevation of piezometric surface at distance r
h_o	elevation of piezometric surface at r_o (original elevation of piezometric surface)
$h_o - h$	drawdown due to pumping
r	distance from center line of well, measured radially outward
r_o	radius of influence of well (distance at which drawdown ≈ 0)
r_w	radius of well at water surface
$\frac{dh}{dr}$	slope of piezometric surface at radius r
A	cross-sectional area of aquifer normal to flow
K	hydraulic conductivity
q	specific discharge (Darcy velocity)
Q	water discharge (pumping or recharge rate)

The drawdown of the piezometric surface results from the reduction of pressure in the aquifer due to pumping.

Assumptions:

a. aquifer is homogeneous and isotropic, and of infinite areal extent (so that boundary effects can be ignored)

b. the well penetrates the entire thickness of the confined aquifer, so that flow in the aquifer is horizontal (i.e., has no upward or downward components.)

c. flow to the well is steady (i.e., flow rate does not vary with time)

d. size of cone of depression of the piezometric surface is constant (this is a corollary of c above)

Derivation:

a. consider a cylinder of aquifer of radius r and height b around the well

b. applying Darcy's Law, the rate of flow to the well is given by:

$$Q = Aq \qquad \text{where} \qquad A = 2\pi rb \qquad q = K\frac{dh}{dr} \qquad \text{hence}$$

$$Q = 2\pi rbK\frac{dh}{dr} \tag{1}$$

Note that because flow is steady and the cone of depression is not expanding, the rate of flow must be the same as the pumping rate and is a constant, i.e., Q = constant.

c.rearranging (1) and then integrating, we get:

$$dh = \frac{Q}{2\pi bK}\frac{dr}{r} \tag{2}$$

$$\int_{h_w}^{h_o} dh = \int_{r_w}^{r_o} \frac{Q}{2\pi bK}\frac{dr}{r} = \frac{Q}{2\pi bK}\int_{r_w}^{r_o}\frac{dr}{r} \tag{3}$$

$$h_o - h_w = \frac{Q}{2\pi bK}\ln\frac{r_o}{r_w} \tag{4}$$

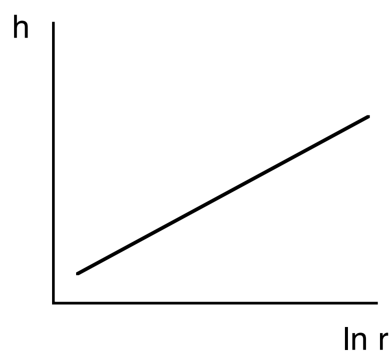
d. rearranging (4)

$$Q = 2\pi bK\frac{h_o - h_w}{\ln\left(\frac{r_o}{r_w}\right)} \tag{5}$$

e. Generalizing to any radius r:

$$\mathbf{Q = 2\pi bK\frac{h_o - h}{\ln\left(\frac{r_o}{r}\right)}} \qquad \text{This is the equilibrium or Theim equation} \tag{6}$$

Note that for any given value of Q, b, and K, this equation implies a linear relationship between h and ln r:

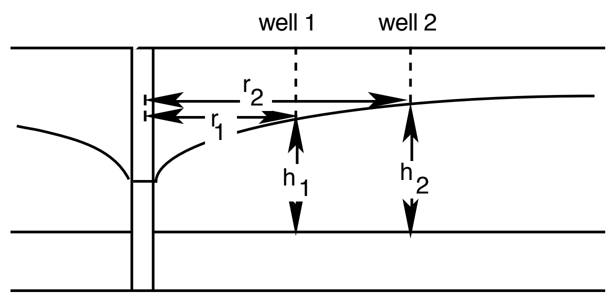


Pumping Test to Determine K

Solve equation (6) for K:

$$K = \frac{Q}{2\pi b\ (h-h_w)} \ln \frac{r}{r_w} \qquad (7)$$

If we drill two observation wells at some distance from the pumping well, we can get measurements (h_1,r_1) and (h_2,r_2).



Since h and ln r have a straight-line relation, we can substitute h_1 and h_2 for h and h_w, and r_1 and r_2 for r and r_w. That is:

$$K = \frac{Q}{2\pi b\ (h_2-h_1)} \ln \frac{r_2}{r_1} \qquad (8)$$

The major difficulty with this method is the assumption of constant cone-of-depression size. In reality, the drawdown cone expands and becomes larger with time. However, drawdown will stabilize after a while and change only very slowly. Thus to make such a test, the well must be pumped at a constant rate for long enough for drawdown to become essentially constant.

Despite the assumptions and restrictions mentioned above, the Thiem relation has been widely used for determinations of hydraulic conductivity.

2. HOMOGENEOUS ISOTROPIC UNCONFINED AQUIFER, STEADY-STATE FLOW, FULLY-PENETRATING WELL